constant temperature of the rotating sphere and T_o. If the problem is restricted to a given fluid, the substance variables N_{Pr} and the quotient parameters will be fixed and the problem reduced to the relation

$$f_{\text{Newt}}\left[N_{Nu}, N_{Re}, N_{Br}, \left(\frac{\partial\eta}{\partial T}\right)_o \cdot \Delta T/\eta_o\right] = 0 \quad (3)$$

Experiments on elastoviscous liquids have shown (*8* to *10*) that the corresponding process in a non-Newtonian medium is coupled with a considerably more complicated flow pattern. However, the heat transport in this case is given by the following relation:

$$f_{n\text{-Newt}}\left[\frac{hd}{K_o}, \frac{\rho_o nd^2}{H_o}, \frac{H_o n^2 d^2}{k_o \cdot \Delta T}, \left(\frac{\partial H}{\partial T}\right)_o \cdot \Delta T/H_o,\ n\theta_o\right] = 0$$

which in comparison with the Newtonian case shows only one additional variable. The function $f_{n\text{-Newt}}$ is determined by the rheological behavior of the medium.

NOTATION

c = specific heat
d = characteristic length
D = rate of shear
f = function
h = heat transfer coefficient
k = thermal conductivity
K = rheological parameter
m = rheological parameter
n = number of revolutions
N_{Br} = Brinkman number
N_{Nu} = Nusselt number
N_{Pr} = Prandtl number
N_{Re} = Reynolds number
P = power
S = symbol for set of pi-variables
T = temperature

Greek Letters

η = viscosity
H = rheological parameter
θ = rheological parameter
π_i = dimensionless variables
ρ = density
τ = shear stress
ψ = compliance

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Laminar Flow of Two Immiscible Liquid Falling Films

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The problem of steady state laminar gravity flow over a flat surface of two immiscible liquids involves the simultaneous solution of the equations of motion for each liquid. Consider the region to be sufficiently far from the ends of the wall that the entrance and exit disturbances are negligible. Denoting the liquid which adheres to the wall as 1 and the outer liquid as 2, the equations of motion and boundary conditions for Newtonian fluids are

$$\mu_i \frac{d^2 v_i}{dx^2} = -\rho_i g_y \quad (1)$$

$$x = 0, \quad v_1 = 0 \quad (2)$$

$$x = \delta_1, \quad v_1 = v_2 \quad (3)$$

$$x = \delta_1, \quad m\frac{dv_1}{dx} = \frac{dv_2}{dx} \quad (4)$$

$$x = \delta_1 + \delta_2 = \delta_T, \quad \frac{dv_2}{dx} = 0 \quad (5)$$

The solution to Equation (1) is a parabola for each liquid. Introducing the boundary conditions stated in Equations (2) to (5) gives the local velocities

$$v_1 = \frac{\rho_1 g_y}{2\mu_1}\left[\left(\frac{2\delta_2}{r} + 2\delta_1\right)x - x^2\right] \quad (6)$$

for $0 \leq x \leq \delta_1$, and

$$v_2 = \frac{\rho_2 g_y}{2\mu_2}\left[\left(\frac{r}{m} - 1\right)\delta_1^2 + 2\delta_1\delta_2\left(\frac{1}{m} - 1\right) + 2\delta_T\, x - x^2\right] \quad (7)$$

for $\delta_1 \leq x \leq \delta_T$.

The average velocity in each film is given by

$$\bar{v}_i = \int v_i dx / \int dx \quad (8)$$

where the integration is over the respective film thickness. Substituting Equations (6) and (7) into Equation (8) and integrating yields

$$\bar{v}_i = \frac{\rho_i g_y \delta_i^2}{3\mu_i} G_i \quad (9)$$

where

$$G_1 = 1 + 3/(2\ r\varphi) \quad (10)$$

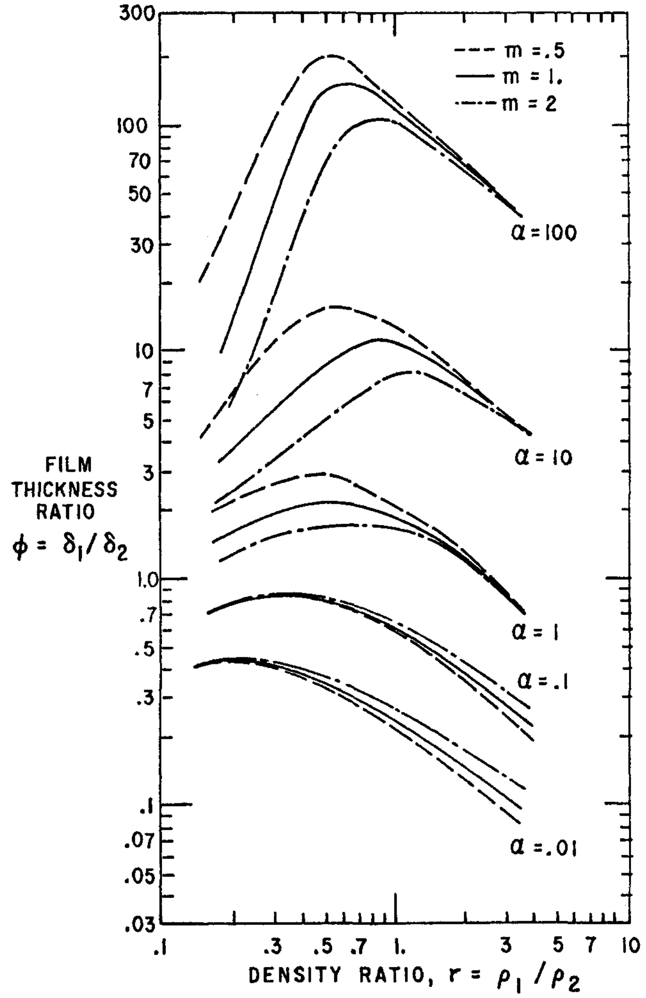

Fig. 1. Film thickness ratio for two immiscible falling films in laminar flow.

$$G_2 = 1 + 3\varphi/m + 3r\varphi^2/m \qquad (11)$$

The mass flow rates per unit width, $\Gamma = \bar{v}\,\delta\rho$, are

$$\Gamma_i = \frac{\rho_i^2 g_y \delta_i^3}{3\mu_i} G_i \qquad (12)$$

and the ratio of the film thickness from Equations (10), (11), and (12) is given by

$$\varphi^3 + 1.5\,[(m\gamma)^{2/3}\, r^{-7/3} - \gamma r^{-1}]\,\varphi^2 - 3\, r^{-2}\varphi - (m\gamma) r^{-2} = 0 \qquad (13)$$

Thus, the film thickness ratio is a function of the ratios of densities, viscosities, and mass flow rates but not of the gravitational force.

Equation (13) was solved numerically, using a digital computer, for a number of combinations of physical properties and flow rates. The results are shown in Figure 1. For given mass flow rates, the ratio of the film thicknesses passes through a maximum at a given density ratio. The viscosity ratio contribution reverses with increased density ratio.

Figure 1 or Equation (13) may be used to determine the average velocities or the mass flow rates from Equations (9) to (12). On the other hand, if the mass flow rates are known, one can calculate the individual film thicknesses and the average velocities. In addition to the above assumptions, the analysis neglects the effects of surface tension.

This analysis was developed in conjunction with a study of film-by-film condensation of immiscible liquids. The subsequent heat transfer treatment parallels that of Nusselt.

ACKNOWLEDGMENT

The Computer Science Center at the University of Maryland provided digital machine time supported in part by the National Aeronautics and Space Administration.

NOTATION

g_y = gravitational force in y direction
G_i = correction in Equation (9) for component i
m = μ_1/μ_2, viscosity ratio
r = ρ_1/ρ_2, density ratio
v_i = velocity of component i
x = distance away from wall

Greek Letters

δ_i = film thickness component i
γ = Γ_1/Γ_2, ratio of mass flow rates
Γ_1 = mass flow rate per unit width for component i
φ = δ_1/δ_2, film thickness ratio
ρ_i = density of component i
μ_i = viscosity of component i

Free Volume Theory for Self-Diffusivity of Simple Nonpolar Liquids

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In a recent paper (*1*), hereafter referred to as I, it has been shown that the Macedo-Litovitz equation (*2*),

$$\eta = A_o T^{1/2} \exp\left(\frac{V_o}{V - V_o}\right) \exp\left(\frac{E_v}{RT}\right) \qquad (1)$$

provides a good description of the effects of temperature and pressure on liquid viscosity of simple liquids for $\rho \geqq 2\rho_c$, provided that the temperature dependence of V_o and the density dependence of E_v are accounted for. By arguments analogous to those of Macedo and Litovitz, a similar equation may be written for the self-diffusion coefficient,

$$D = B_o T^{1/2} \exp\left(-\frac{V_o(T)}{V - V_o(T)}\right) \exp\left(-\frac{E_v(V)}{RT}\right) \qquad (2)$$

In this paper we test Equation (2) for liquids composed of monatomic and quasispherical polyatomic molecules. According to Equations (1) and (2), V_o and E_v should be the same for viscosity and self-diffusion coefficient at a particular temperature and density. The quantity V_o was therefore obtained from Equations (3), (4), and (5a) of paper I (assuming $n = 12$ in the 6, n potential), and E_v was obtained from curve A of Figure 4 in paper I. The remaining parameter B_o was evaluated by comparing experimental data with Equation (2), using the computer.

Tables 1 and 2 show the data sources and parameters for the fluids studied, while Figure 1 illustrates the agreement for methane. Values of ϵ/k, T_m, V_m and V_c used were those of table 2 in paper I. Unfortunately no